ACTIVITY

2

NAME ______________________________

Sorting Sequences

Find the next three entries in each of these sequences.

1\. 5, 10, 15, 20, 25, ____, ____, ____

2\. 9, 16, 23, 30, 37, ____, ____, ____

3\. 51, 48, 45, 42, 39, ____, ____, ____

4\. 100, 94, 88, 82, 76, ____, ____, ____

5\. 0, 2, 6, 12, 20, ____, ____, ____

6\. 4, 5, 7, 10, 14, ____, ____, ____

7\. 999, 998, 996, 993, 989, ____, ____, ____

8\. 70, 68, 64, 62, 58, ____, ____, ____

9\. 14, 9, 14, 9, 14, ____, ____, ____

10\. 1, 11, 10, 20, 19, ____, ____, ____

11\. 1, 3, 2, 4, 3, ____, ____, ____

12\. 500, 450, 458, 408, 416, ____, ____, ____

13\. 1, 8, 22, 43, 71, ____, ____, ____

14\. 50, 54, 62, 74, 90, ____, ____, ____

15\. 1, 1, 2, 3, 5, 8, ____, ____, ____

16\. 1, 3, 9, 27, 81, ____, ____, ____

17\. 1, 3, 4, 7, 11, 18, ____, ____, ____

18\. 2, 8, 32, 128, ____, ____, ____

19\. 48, 24, 12, ____, ____, ____

20\. 1, 8, 5, 12, 9, 16, ____, ____, ____

21\. 1, 4, 9, 16, 25, ____, ____, ____

22\. 1, 11, 111, 1111, ____, ____, ____

23\. 2, 2, 3, 4, 6, 9, 14, ____, ____, ____

24\. O, T, T, F, F, S, S, E, ____, ____, ____

ACTIVITY **1** NAME ______________________________

Fibonacci Figuring

Eight hundred years ago there lived a mathematician Leonardo of Pisa, who was called Fibonacci because he was the son of Bonacci. He was the first person to write down an important list of numbers that has since been named for him. You can easily construct his list of numbers.

This is the rule:

Each number is the sum of the previous two numbers in the list.

- Begin with the number 1, the most basic of all numbers.
- Add that first number (1) to the previous number in the list (in this case zero, since there is no previous number). **1 + 0 = 1**
- Add the previous two numbers to get the next number. **1 + 1 = 2**
- Add the previous two numbers to get the next number. **2 + 1 = 3**
- To continue, create each number by adding the previous two numbers.

List

1

1

2

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___

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The numbers in this list are the *Fibonacci numbers.*

When they are listed in this order, they are the *Fibonacci sequence.*

After you reach the bottom of this sheet, continue the Fibonacci sequence as long as you like on the back. Look at your numbers and find interesting patterns and relationships. Then, answer these questions.

1. What is the 5th Fibonacci number? _____

What is interesting about this?

2. What is the 12th Fibonacci number? _____

What is interesting about this?

3. How many Fibonacci numbers are there? _____

4. What seems to be the ratio of odd Fibonacci numbers to even Fibonacci numbers?

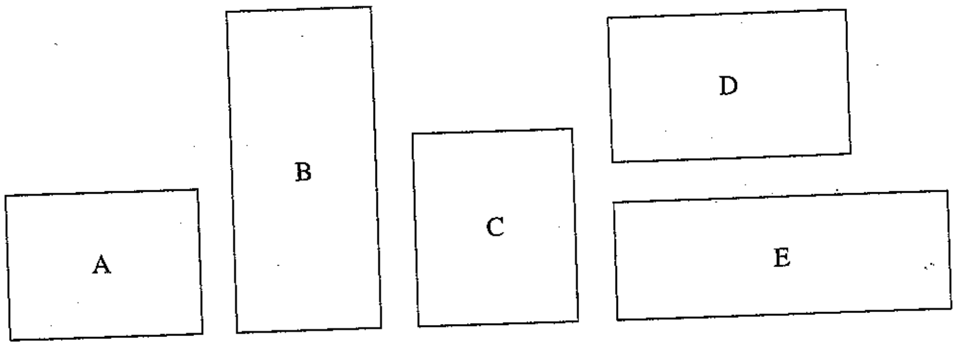
5. What is the 25th Fibonacci number? _____

PROJECT 1

NAME ____________________

Rectangle Beauty Contest

Ask your friends, relatives, and neighbors to pick from the set of rectangles the one they consider the most visually pleasing. Ask, "Which is your favorite rectangle?"



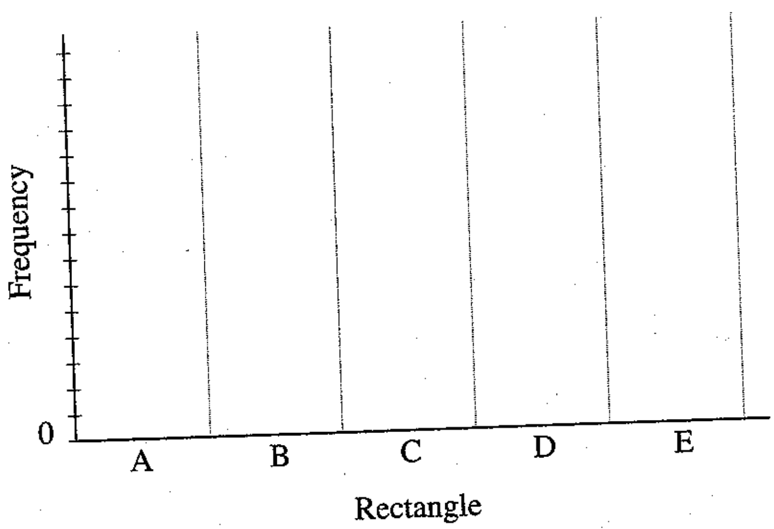
Record your findings, then record the number of tally marks in the frequency column.

Tally (use slashes ////\ /)	Frequency
A	
B	
C	
D	
E	

Make a graph (also called a histogram) using the data you gathered. On the left, use the numbers for the frequency that will fit your data.

Write a summary of your findings. Include the winning rectangle and its dimensions. Combine your findings with your classmates. What conclusions can you draw?

Favorite Rectangle



pieces in which ideas of perspective or symmetry are important.

3. **Escher.** Look on the Web for art by M. C. Escher. Choose one of his works and write a short essay about his use of perspective in the piece.

4. **Penrose Tilings.** Many Web sites are devoted to Penrose tilings (aperiodic tilings). Visit one such site and learn more about the nature and uses of these tilings. Write a short essay describing your findings.

UNIT 10D

PROPORTION AND THE GOLDEN RATIO

The senses delight in things duly proportioned.

—ST. THOMAS AQUINAS (1225–1274)

In Unit 10C, we studied how mathematics enters into art through the ideas of symmetry and perspective. In this unit, we turn our attention to the third major mathematical idea involved with art: proportion.

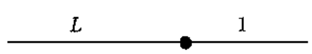
The importance of proportion was expressed well by astronomer Johannes Kepler (1571–1630):

> *Geometry has two great treasures: one is the theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to gold; the second we may name a precious jewel.*

Kepler's statement about *the division of a line into extreme and mean ratio* describes one of the oldest principles of proportion, which dates back to the time of Pythagoras (c. 500 B.C.) when scholars asked the following question: How can a line segment be divided into two pieces that have the most visual appeal and balance?

Surprisingly, although this was a question of beauty, there seemed to be general agreement on the answer. Suppose a line segment is divided into two pieces, as shown in Figure 10.60. We will call the length of the long piece L and the length of the short piece 1.

FIGURE 10.60



The Greeks claimed that the most visually pleasing division of the line had the property that the ratio of the length of the long piece to the length of the short piece is the same as the ratio of the length of the entire line segment to the length of the long piece. That is,

$$\frac{L}{1} = \frac{L+1}{L}$$

This statement of proportion can be solved (see Exercise 8) to find that L has a special value, denoted by the Greek letter ϕ (pronounced "fie" or "fee"), which is

$$\phi = \frac{1+\sqrt{5}}{2} = 1.61803\ldots$$

The number ϕ is more commonly called the **golden ratio** (or *golden section*); it is also sometimes called the *divine proportion.* Note that the golden ratio is an irrational number often approximated as 1.6, or $\frac{8}{5}$. Figure 10.61 shows that, for any line segment

by the way . . .

The Greek letter ϕ is the first letter in the Greek spelling of Phydias, the name of a Greek sculptor who may have used the golden ratio in his work.

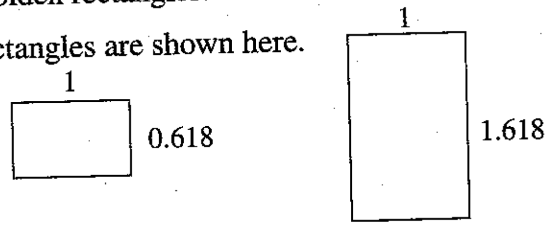
ACTIVITY 12

NAME ______________________

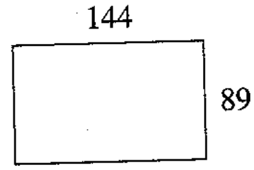
Beautiful Problems

Certain rectangles are especially pleasing to the eye. They are not too wide and not too long. They are called golden rectangles.

The proportions of such rectangles are shown here.



Rectangles whose side measurements are consecutive Fibonacci numbers larger than 5 are also golden—the numbers provide an easy way to achieve the golden ratio. The Fibonacci numbers are 1, 1, 2, 3, 5, 8, and so on, where each number is the sum of the previous two numbers.



Keep this in mind as you answer these questions.

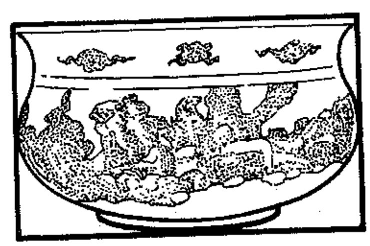
1. An artist wishes to paint in the shape of the golden rectangle. If the paper is 21 inches wide, how long should it be? ____

   If it is 89 centimeters long, how wide should it be? ____

2. A standard piece of computer paper is $8\frac{1}{2}$ inches wide and 11 inches long. Is it a golden rectangle? ____ too wide? ____ too long? ____

3. A sketch of the front view of the Parthenon, which has golden proportions, is 45 centimeters long. How high should it be?

   Hint: $\frac{\text{small}}{\text{large}} = \frac{0.618}{1}$



4. A bowl dating to the Ching dynasty in China has an 8-inch diameter at the top. How deep is it if it exhibits golden proportions? ____

5. A standard paperback novel is a golden rectangle 10.5 centimeters wide. How high is a standard paperback? ____

6. A standard, single light-switch plate is 11.5 centimeters long. If it is a golden rectangle, how wide is it? ____

7. A carpenter wishes to install some doors to a patio that are visually pleasing. If the doors need to be $6\frac{1}{2}$ feet high, how wide should they be? ____

8. A Navajo rug being woven is 5.5 feet wide. To have golden proportions, the person weaving it should stop after how many feet of length? ____